mation for the ground state. In view of the previous studies,[5,7] the use of a Bessel-like wave function for $r<R_0$ and a modified hydrogen-like function for $r>R_0$ may lead to a ground-state energy lying below the one predicted by using just a hydrogen-like function.

For the excited states, the modification of the Bessel-like wave function leads to a negligible change in the energy levels, since the F-center electron spends most of its time in the Coulomb-like potential region. Hence, the choice of a hydrogen-like function for the $2p$ or the other higher state is expected to be a good approximation.

Consequently, the approach to the problem of the trapped electron, which includes phonons and phonon-electron interactions does raise the predicted position of the energy level of a trapped electron relative to previous calculations and can bring the position of the energy level into line with recent experiment,[8] due to a small quantity of positive energy contributed by the phonons.

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Field and Angular Dependence of Critical Currents in Nb_3Sn*

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Measurements have been made on the critical current of Nb_3Sn vapor-deposited strips on ceramic material. The Nb_3Sn was obtained from vapor deposition and is single phase without any preferred orientation in the plane of the strip. Critical currents were obtained at 4.2°K as a function of field up to 20 000 G, and as a function of the angle between the field and the current axis. Below 10 000 G a sharp rise in critical current by about a factor of two was observed for longitudinal fields, and a gradual decrease in critical current for transverse fields. Above 10 000 G the field and angular dependence is in good agreement with the predictions of the Lorentz force model of Kim *et al.* Below 10 000 G, the field dependence can be associated with a transition from an inhomogeneous to homogeneous current distribution with consequent lower local critical current densities. Experimental results are also reported for field shielding measurements on similar material in cylindrical form. Excellent agreement is obtained in both magnitude and field dependence with the strip data.

THAT the magnetic behavior of hard superconducting tubes in a longitudinal field H, can be described in terms of a single model depending on the concept of a critical state,[1] and a field-dependent critical current density, j_c, where

$$j_c=\alpha/(H+B_0), \quad (1)$$

has been shown by Kim, Hempstead, and Strnad.[2] In (1), α and B_0 are constants depending on the temperature and the properties of the medium. Anderson[3] has shown that (1) can be obtained in terms of flux penetration through the specimen[4,5] and the Lorentz force exerted on a flux bundle by the current. When a pinning force on a flux bundle is exceeded, either the phenomena of flux creep or quenching can occur.[1,6]

In the magnetization experiments of Kim *et al.*[2] the current and field are perpendicular to each other, and the current is internally generated. An important question concerns the extension of the above ideas to linear geometries where current is externally supplied. Apart from the practical significance of these geometries there

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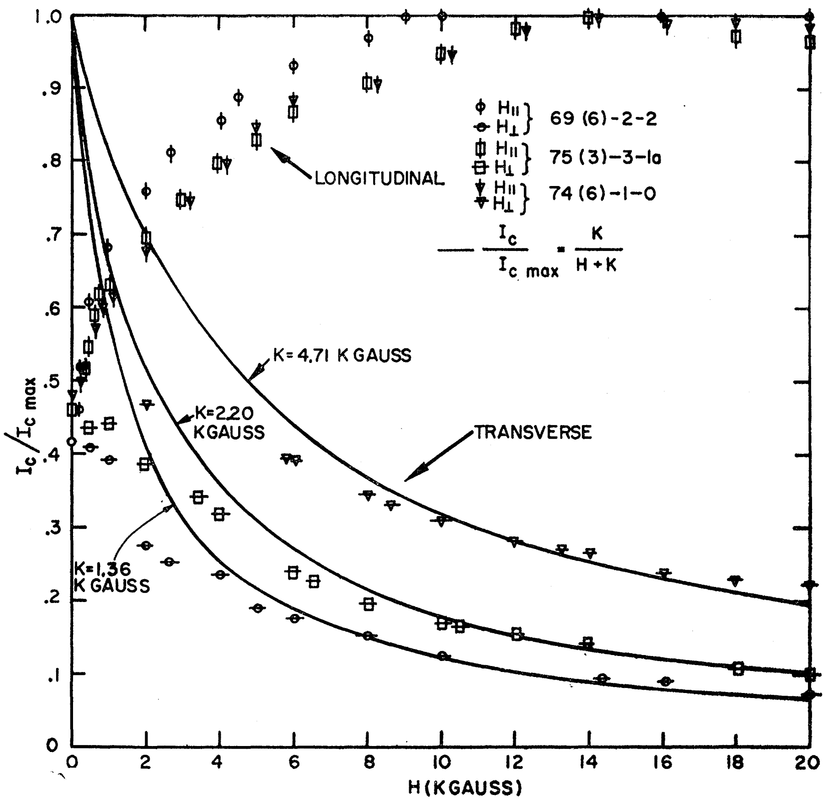

FIG. 1. Quenching current, for Nb_3Sn strips normalized to maximum in longitudinal direction as a function of field for fields transverse and longitudinal to the specimen axis. For both configurations the quenching current is reproducible, and independent of whether field or current is applied first, as long as the transition is made from the superconducting state.

is the additional advantage that the angle between the applied field and the current can be changed furnishing a strong test of the Lorentz force model. We have made measurements of the critical currents of Nb_3Sn strips as a function of magnetic field and the angle between the current and the applied field. We can interpret the field and angular dependence of the critical currents in terms of a simple extension of the Lorentz force model of Anderson.[3] Consistent with this explanation of the critical current, we find agreement between the current densities obtained from the magnetic behavior of a tube of Nb_3Sn and the current densities obtained for similar material in transverse fields where the current is externally supplied. Finally, we have observed for all samples what appears to be new phenomena: a sharp rise in the critical current for low longitudinal fields, and subsequent field independence up to the highest field applied (20 000 G). We believe the field enhancement of the critical current, (by as much as a factor of two) is evidence for an inhomogeneous current distribution in Nb_3Sn at low fields as is observed for soft superconductors.

TABLE I. Characteristics of Nb_3Sn samples.

Sample	T_c[a] (°K)	ΔT_c (°K)	Width (10^{-2} cm)	Thickness (10^{-4} cm)	J_{max} (10^5 A/cm²)
69(6)–2–2	18.1	2.9	2.3	32.0	15[c]
74(6)–1–0	17.6	0.8	1.1	20.0	32[c]
75(3)–3–1[a]	16.8	0.8	1.4[b]	27.0	24[c]
77(6)–1–1	18.0	0.2	1.8[b]	30.0	12[d]
77(6)–2–0	18.0	0.2	1.4	23.0	19[d]

[a] Start of ac transition.
[b] Nonrectangular cross section.
[c] 20 kG.
[d] 7.4 kG.

In this letter we present critical current data on five Nb_3Sn strips and one Nb_3Sn cylinder. All samples of Nb_3Sn that we have examined show similar characteristics; the samples presented were chosen for the range of behavior displayed. The samples were obtained from material obtained by a gas-phase deposition technique for Nb_3Sn on polished ceramic flats.[7,8] Photomicrographic and x-ray analyses showed the strips to be single phase with a columnar structure perpendicular to the plane of the strip, but with no preferred orientation in the plane of the strip. The stoichiometry was close to that for Nb_3Sn. Each flat was formed by sand blasting into dumbbell shaped strips whose dimensions are given in Table I. The transition temperatures determined by an ac inductance method[9] are also given in Table I.

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Critical currents were determined by the first sign of voltage on an amplifier whose noise level was 0.2 μV. Potential and current contacts were made to nickel or copper plated regions on the strip. The length of the specimen between current contacts ranged from 4 to 7 mm. Indium solder was used for contacts, which at the temperature of the present experiment were nonsuperconducting (4.22°K). Flux-shielding measurements were made on cylinder No. 77 in the manner of Kim *et al.*[2] These measurements will be described in another paper.[10] Only the presently relevant results will be given here, namely, the derived critical current density as a function of field.

Figure 1 shows the critical current as a function of field for both transverse and longitudinal configurations for several samples. The data are normalized to the highest current density obtained in a longitudinal field. Figure 2 shows the angular dependence for the same specimens at constant field, again normalized to the longitudinal configuration. The curves in Figs. 1 and 2 are theoretical, based on a model to be discussed subsequently. Table I gives the maximum current density obtained for each sample. The scatter in values can be either associated with geometrical uncertainties or slight compositional variations.[9]

We associate the sharp rise in quenching current for longitudinal fields less than 8000 G with a transition from a superficial to a uniform current distribution. In zero-field the current is largely confined to within a penetration depth[11] (0.3 μ) of the edge. It is clear that such a distribution should exhibit larger edge fields than the uniform distribution, and hence, should have considerably higher current densities at the edge and corner of the strip.[12] It is a basic assumption of the present paper that the motion of flux in Nb_3Sn at the high levels of current density leads to quenching through local heating.[6] For this case, the sample will therefore quench in a Silsbee-type mechanism determined by a critical current density at the edge of the strip. The rise in critical current with longitudinal field is thus not a function of any preferred orientation in the specimen, but is due to the homogeneous distribution established by the field with consequent lower local current density. On the basis of the Lorentz force model we expect the longitudinal field critical current for high fields to be independent of external field, as is seen in Fig. 1.

The second assumption of the present letter is thus that there are two regions of external field to consider. Below about 8000 G flux penetration is not yet complete; the current distribution is still somewhat inhomogeneous, and the superconducting to normal transition dominated by a Silsbee-type mechanism. Above

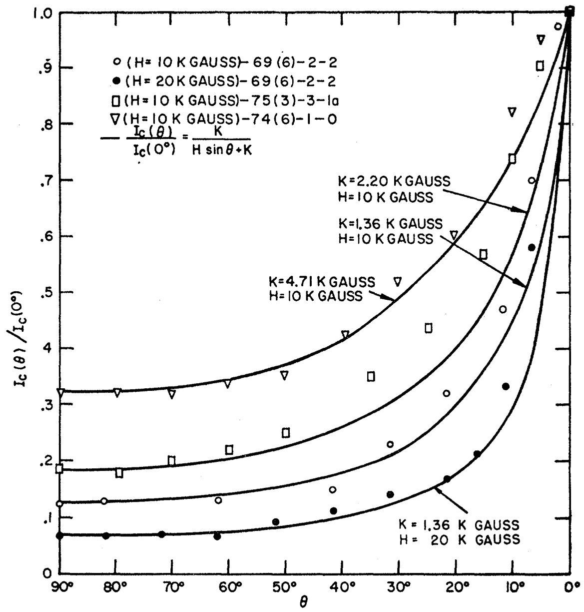

FIG. 2. Quenching current, for Nb_3Sn strips shown in Fig. 1, normalized to maximum in longitudinal direction as a function of angle at constant field.

8000 G one assumes essentially complete penetration and that the Lorentz force arguments of Anderson[3] apply.

The simplest modification of (1) to take into account the nonperpendicular field of the present experiment is

$$j_c(H \sin\theta + B_0) = \alpha , \quad (2)$$

where θ is the angle between the field and the axis of the strip. Thus, if we assume the critical current, I_c, to be parallel to the axis we obtain

$$I_c(\theta)/I_c(0°) = K/(H \sin\theta + K) , \quad (3)$$

where K and $I_c(0)$ are functions of α, B_0 and the geometry of the strip. If one includes the field produced by the current we can obtain an expression of the form

$$I_c(\theta)/I_c(0°) = (1/K')\{-H \sin\theta + [H^2 \sin^2\theta + (K')^2]^{1/2}\} . \quad (4)$$

Equation (4), although it corresponds to a physically distinct situation, differs by about 10% from (3) over the entire range in field. Given the crudeness of the model we will use (3) rather than (4) in the analysis of the data.

Equation (3) predicts an I_c independent of field in the longitudinal configuration. The prediction agrees with experiment for fields above about 8000 G. For the transverse case we can determine the constant K from the maximum anisotropy shown in Fig. 2 at a given field. Curves based on this "experimental" value of K are shown in Fig. 1 and again the agreement is good above 8000 G.

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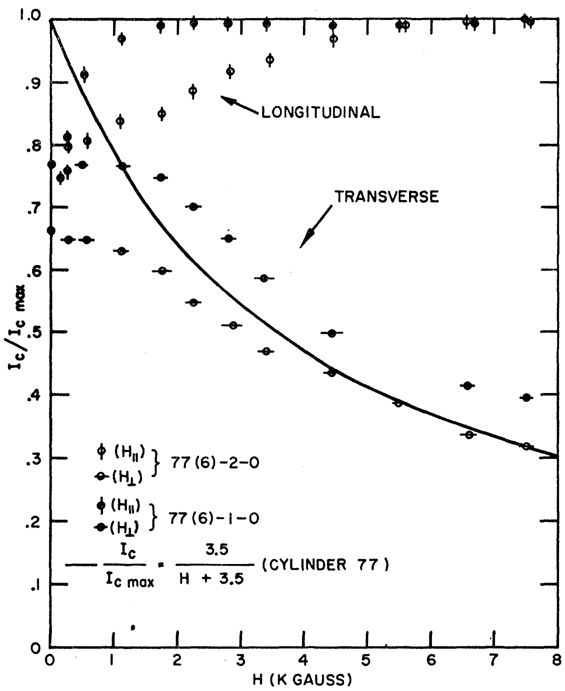

FIG. 3. Comparison between normalized critical current densities obtained for Nb_3Sn strips and current densities obtained from magnetization of Nb_3Sn cylinder. Both the strips and the cylinder were prepared in the same deposition run (77).

The angular variation predicted by (3) is shown by the solid lines in Fig. 2, for field values shown in the figure. The agreement is good, and could be improved if a slight angular variation in the current density about the strip axis were taken into account in (3). Indeed, as shown by sample 69(6)–2–2 in Fig. 2, the field variation of the angular dependence is also in good agreement with (3) for $H=10\,000$ G and $H=20\,000$ G.

For comparison with the work of Kim *et al.*,[2] the data of Fig. 3 are given. This figure shows the transverse and longitudinal field dependence for two strips from deposition run 77. The absolute values of critical current densities for the strips at 7500 G are in fair agreement with each other, within geometrical uncertainties. More significant, they are in good agreement with the current density at this field derived from magnetization measurements on cylinder 77, prepared in the same deposition run. The smooth curve in Fig. 3 is the curve obtained from (2) determined by the angular variation of 77(6)–2–0; it is identical with the field dependence obtained from the magnetization[10] i.e.,

$$J_c=\frac{6.35\times10^6}{H+3.5}\ \text{A/cm}^2. \tag{5}$$

It is a striking confirmation of the ideas of Kim *et al.*[2] and Anderson[3] that the distinct experiments; transverse field dependence of critical current, constant field angular variation of critical current, and flux shielding by a cylindrical tube, should agree so well in magnitude and functional dependence with the predictions of the Lorentz force model.

Similar angular dependences as in Fig. 2 have been observed previously for other hard superconductors[13] but the material was strongly cold worked, and presumably had a preferred orientation. Some of the models described by these authors have an angular dependence similar to (3), but do not give the proper field dependence.

A maximum in critical current with applied field has been observed previously, but appeared to be associated with preferred orientation, and occurred at considerably higher fields than the rise in critical current observed in the present experiments.[14–16]

The significance of the present results appears to be the following: first, evidence for Silsbee-type current quenching in single phase Nb_3Sn at low fields; second, evidence for the ability of a longitudinal field to establish a uniform current distribution in Nb_3Sn that is field-independent; third, evidence for the role played by the Lorentz force, not only in limiting the critical current in transverse fields, but also in determining the angular variation of critical current with applied field; fourth, a demonstration of the agreement between critical current densities obtained by the flux shielding technique of Kim *et al.*,[2] and direct measurements of critical currents. Finally, the present work again indicates the possibility of enhanced current capacities for solenoid configurations that minimize Lorentz forces due to the solenoid field.

Work is continuing on the temperature dependence of these effects, and their relation to other superconducting properties of Nb_3Sn.

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